Bose Einstein Scholarship Test

An endeavour of International Research Scholars and Mentors with JMMC Research Foundation

Sample Question for Class - Major

1. Let F be a field. Given below are six statements about F.
   1. F is a skew field
   2. F is a group with respect to multiplication
   3. F is an integral domain
   4. F has zero divisors
   5. F has no zero divisors
   6. Only ideals of F are $\{0\}$ and itself In which of the following options all the statements are correct?

   (a) 1,2,3 (b) 1,3,5 (c) 2,4,6 (d) 4,5,6

2. Let $f(x,y)=\ln\sqrt{x+y}$ and $g(x, y)=\sqrt{x+y}$. Then the value of ∇^2 (fg) at (1, 0) is

   (a) $-\frac{1}{2}$ (b) 0 (c) $\frac{1}{2}$ (d) 1

3. Let $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 2 & 10 & 8 & 5 & 9 & 3 & 6 & 11 & 4 & 12 & 1 & 7 \end{pmatrix}$

   the cardinality of the orbit of 2 under σ is

   (a) 3 (b) 6 (c) 9 (d) 12

4. The general solution of the differential equation

   $y''(x) - 4y'(x) + 8y(x) = 10e^x \cos x$ is

   (a) $e^{2x}(k_1 \cos 2x + k_2 \sin 2x) + e^x(2\cos x + \sin x)$

   (b) $e^{2x}(k_1 \cos 2x + k_2 \sin 2x) + e^x(2\cos x - \sin x)$

   (c) $e^{-2x}(k_1 \cos 2x + k_2 \sin 2x) - e^x\, 2\cos x - \sin x)$

   (d) $e^{-2x}(k_1 \cos 2x + k_2 \sin 2x) + e^x(2\cos x + \sin x)$

5. Let $f(x) = x^2 + 1$, $g(x) = x^3 + x^2 + 1$ and

   $h(x) = x^4 + x^2 + 1$. Then

   (a) $f(x)$ and $g(x)$ are reducible over Z_2

   (b) $g(x)$ and $h(x)$ are reducible over Z_2

   (c) $f(x)$ and $h(x)$ are reducible over Z_2

   (d) $f(x)$, $g(x)$ and $h(x)$ are reducible over Z_2

6. The surface area of the solid generated by revolving the line segment $y = x + 2$ for $0 \leq x \leq 1$ about the line $y = 2$ is

(a) $\sqrt{2\pi}$ (b) 2π (c) $2\sqrt{2\pi}$ (d) 4π

7. The differential equation $2ydx - (3y - 2x)\, dy = 0$ is

(a) exact and homogeneous but not linear

(b) homogeneous and linear but not exact

(c) exact and linear but not homogeneous

(d) exact, homogeneous and linear

8. If $\vec{u}(t) = u_1(t)\, i + u_2(t)\, j + u_3(t) k$ is a unit vector and $\frac{d\vec{u}}{dt} \neq 0$, then the angle between $\vec{u}(t)$ and $\frac{d\vec{u}}{dt}$ is

(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

9. The order of 2 in the field $\mathbb{Z}_{29}$ is

(a) 2 (b) 14 (c) 28 (d) 29